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Some Remarks on ω and φ Meson Widths and R-Conjugation Invariance in Strong Interaction

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It is pointed out that partial widths of two neutral vector mesons, ω and φ, into three pions provide some information for or against R-conjugation invariance in strong interaction. A particular model is considered in detail, in which ω and φ mesons are R conjugation invariantly associated with baryonic charge and hypercharge currents, respectively. This model is shown to explain consistently various phenomena involved in production and decay processes of these vector mesons.

1. INTRODUCTION

ONE of the current interests of the strong interaction theory will be to look for some higher symmetries among strongly interacting particles. Many attempts so far have been devoted both theoretically and experimentally to finding such possible symmetries of strong interaction.[1]

It has been shown through these works that SU(3)[2] or G_2,[3] as well as some discrete symmetry such as hypercharge reflection or R-conjugation invariance, seem to be the most promising ones.

The purpose of this note is to point out that widths of two neutral vector mesons, ω and φ, when combined with dynamical properties of these mesons, could offer some interesting information for or against R invariance which was originally proposed by Feinberg and Behrends,[4] Sakurai,[5] and recently discussed in detail by Okubo and Marshak.[6]

Experimental observations of ω and φ meson widths seem to suggest that $\Gamma(\omega \rightarrow 3\pi)$ falls around several MeV ($\gtrsim 5$ MeV),[7] while $\Gamma(\varphi \rightarrow 3\pi)$ is very small ($\lesssim 1$ MeV).[8]

On the other hand, several authors[9] have theoretically guessed the ω meson width $\Gamma(\omega \rightarrow 3\pi)$. Their results, however, do not agree with each other but vary from 0.4 to 20 MeV according to the assumptions taken in their calculations. Therefore, we shall not take the conclusions obtained by these authors seriously. Moreover, we shall assume neither SU(3) nor G_2 symmetry in strong interaction, although we always keep these symmetries in mind throughout this paper.

In Sec. 2, a preliminary argument in favor of R' or R'' invariance discussed by Okubo and Marshak is given. In Sec. 3, we examine the partial decay rates of ω and φ mesons into three pions in connection with R-conjugation invariance and, finally, in Sec. 4, a model in which ω and φ mesons are R' or R'' invariably associated with baryonic charge and hypercharge currents, respectively, is proposed and some consequences based on this model are compared with experiments.

2. PRELIMINARIES: $\gamma-3\pi$ INTERACTION

Let us start our discussions by noting that the magnitude of the $\gamma-3\pi$ interaction, which is closely connected with ω or $\varphi \rightarrow 3\pi$ decay interaction, can be estimated by making use of neutral pion decay into two gammas. Previously, we calculated[10] this $\gamma-3\pi$ vertex with the Gell-Mann and Zachariasen model (ρ and ω dominant model) and showed that the magnitude of this vertex should be considerably smaller than that predicted by simple perturbational considerations in order to reproduce the observed neutral pion decay

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[1] *Proceedings of the 1962 International Conference on High-Energy Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962). See also Theoretical Physics—Lectures presented at a seminar at Trieste—International Atomic Energy Agency, Vienna, 1963 (unpublished).

[2] M. Ikeda, S. Ogawa, and Y. Ohnuki, Progr. Theoret. Phys. (Kyoto) **22**, 715 (1959); **23**, 1073 (1960); S. Sawada and M. Yonezawa, *ibid.* **23**, 662 (1960); Y. Yamaguchi, Progr. Theoret. Phys. (Kyoto) Suppl. **11**, 1 (1959); Progr. Theoret. Phys. (Kyoto) **23**, 882 (1960); M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Y. Ne'eman, Nucl. Phys. **26**, 222 (1961); S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 949 (1962); **28**, 24 (1962); Phys. Letters **4**, 14 (1963).

[3] R. E. Behrends and A. Sirlin, Phys. Rev. **121**, 324 (1961); Phys. Rev. Letters **5**, 476 (1960); R. E. Behrends, J. Dreitlein, C. Fronsdal, and B. W. Lee, Rev. Mod. Phys. **34**, 1 (1962); R. E. Behrends and L. F. Landovitz, Phys. Rev. Letters **11**, 296 (1963).

[4] G. Feinberg and R. E. Behrends, Phys. Rev. **115**, 745 (1959).

[5] J. J. Sakurai, Phys. Rev. Letters **7**, 426 (1961).

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[9] G. Feinberg, Phys. Rev. Letters **8**, 151 (1962); M. Gell-Mann, D. Sharp, and W. G. Wagner, *ibid.* **8**, 261 (1962); F. Zachariasen and C. Zemach, Phys. Rev. **128**, 849 (1962); W. Frazer and D. Wong, *ibid.* **128**, 1927 (1962).

[10] K. Kawarabayashi and A. Sato, Progr. Theoret. Phys. (Kyoto) Suppl. **21**, 3 (1962).

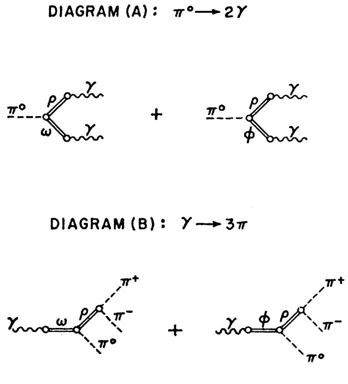

FIG. 1. Diagram (A) shows a model of the neutral pion decay through ρ and $\omega(\varphi)$ meson intermediate states and the corresponding diagram for $\gamma \to 3\pi$ transition amplitude is illustrated in diagram (B). As is easily seen from these diagrams, the ratio of these matrix elements is independent of the vertices corresponding to $\omega\rho\pi$, $\varphi\rho\pi$, $\omega\gamma$, and $\varphi\gamma$.

rate. Our results were[11]

$$\bar{H}^2 \approx 0.1 \bar{H}_{\mathrm{pert}}^2. \tag{1}$$

It is true that we had not taken into account a possible φ-meson effect in our earlier calculations, but we can show that the inclusion of the φ meson does not give any substantial effect for evaluating the magnitude $\bar{H}^2$.[12] The reason simply is that $\omega\rho\pi$ and $\varphi\rho\pi$ vertices appear in the same combination for both matrix elements describing $\gamma \to 3\pi$ and $\pi^0 \to 2\gamma$ processes, so that we can express the latter process in terms of $\bar{H}$ in just the same way as in Ref. 10 (see Fig. 1).

Experimental data on the magnitude of the $\gamma \to 3\pi$ amplitude are still uncertain and vary from one experiment to another.[13] It is to be noted, however, that recent experiments[14] seem to indicate a rather small magnitude of $\bar{H}^2$ compared to $\bar{H}_{\mathrm{pert}}^2$, which is consistent with the foregoing estimation.

In view of these facts, it may be allowed to assume Eq. (1) to be true, which implies, first of all, that the true amplitude $\bar{H}$ should be small compared to $\bar{H}_{\mathrm{pert}}$ or, in other words, the decay rates of $\gamma \to 3\pi$ and $\pi^0 \to 2\gamma$ should be suppressed by an order of magnitude compared to those predicted by simple perturbation calculations. We would like to attribute this fact to some symmetry property of strong interaction.

Of course, there may be many possibilities for reducing these decay rates, but one of the simplest explanations will be to assume the R' or R'' invariance introduced by Okubo and Marshak.[6]

Then, it is easily seen that both $\pi^0 \to 2\gamma$ and $\gamma \to 3\pi$ transitions are forbidden by R' or R'' invariance but not by the usual R invariance.[15] The nonvanishing value of $\bar{H}^2$ may be obtained by taking into account a small R' or R'' invariance violating effect such as the mass difference between nucleon and cascade particle. We can expect according to Eq. (1) that an effectively reducing coefficient β defined by $\bar{H}^2 = \beta \bar{H}_{\mathrm{pert}}^2$ is about 0.1.

3. PARTIAL DECAY WIDTHS OF ω AND φ INTO THREE PIONS

In order to calculate and discuss ω-meson width, one must know something about the coupling constant of the ω meson to the nucleon $f_{\omega NN}$. This coupling constant could be estimated in various ways: for instance, by studying nucleon-nucleon scattering[16] or by production of ω mesons by pions or protons; it is now believed to be large in magnitude.[17] We list in Table I some values obtained by several authors. From Table I we can expect $f_{\omega NN}^2/4\pi \approx 5 \sim 10$, although this

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[11] This value is corrected by using the newly determined π^0 lifetime. Here the amplitude H is defined by

$$\langle 0 | j_\mu{}^s | k^+, k^-, k^0 \text{ out} \rangle = i(8\omega_+\omega_-\omega_0)^{-1/2} \epsilon_{\mu\nu\lambda\sigma} k_\nu{}^+ k_\lambda{}^- k_\sigma{}^0 H^*$$

where k^+, k^-, k^0 are, respectively, four momenta of positive, negative, and neutral pions. From the boson character of pions $H(q^2; s_1, s_2, s_3)$ is a completely symmetric function of the three invariant variables

$$s_1 = -(k^- + k^0)^2,$$
$$s_2 = -(k^0 + k^+)^2,$$
$$s_3 = -(k^+ + k^-)^2,$$

and is also dependent on the squared virtual photon mass $-q^2$. These four variables are connected with the relation

$$s_1 + s_2 + s_3 = 3\mu^2 - q^2.$$

If we calculate the invariant amplitude H in lowest order perturbation theory (nucleon loop current), we get

$$\bar{H}_{\mathrm{pert}} \approx \frac{eg^3}{12\pi^2 m^2}.$$

For detailed discussions, see Ref. 10. We use the same notations and definitions as given in Ref. 10.

[12] This statement is true only if we assume the same parameter α, defined by Eq. (11.54) of Ref. 10, for both ω and φ mesons. This is, however, very plausible since α depends on the final state interactions among the outgoing three pions. We illustrate in Fig. 1 how vertices $\omega\rho\pi$ and $\varphi\rho\pi$ appear in the matrix element of the neutral pion decay.

[13] B. de Tollis, E. Ferrari, and H. Munczek, Nuovo Cimento **18**, 198 (1960); C. S. Robinson, P. M. Baum, and L. Criegee, Phys. Rev. Letters **9**, 349 (1962); J. S. Ball, *ibid.* **5**, 73 (1960); Phys. Rev. **124**, 2014 (1961); W. Alles and D. Boccalette, Nuovo Cimento **27**, 306 (1963); see also Y. Fuji and M. Kawaguchi, Progr. Theoret. Phys. (Kyoto) **28**, 915 (1961); K. Itabashi and T. Ebata, *ibid.* **28**, 915 (1962); M. Monda, *ibid.* **28**, 964 (1962); M. Kawaguchi and H. Yokomi, Progr. Theoret. Phys. (Kyoto) Suppl. **21**, (1962).

[14] J. Pine and M. Bazin, Phys. Letters **5**, 168 (1963); J. P. Burq and J. K. Walker, Phys. Rev. **132**, 447 (1963); J. Pine and M. Bazin, *ibid.* **132**, 2735 (1963).

[15] Perturbation theoretically, this implies that the contributions arising from nucleon and cascade particle loop currents cancel with each other in case of R' or R'' invariance but not in case of R invariance. It will be noted that R and R' invariances can be consistent with SU(3), while R'' invariance can be consistent with global symmetry or G_2, as was pointed out by Okubo and Marshak (Ref. 6). For convenience, we reproduce here definitions of R, R', and R'' conjugations given in Ref. 6:

$$\begin{aligned} R:\quad & p \leftrightarrow \Xi^-,\ n \leftrightarrow -\Xi^0,\ \Lambda \leftrightarrow \Lambda,\ \Sigma^+ \leftrightarrow \Sigma^-,\ \Sigma^0 \leftrightarrow \Sigma^0, \\ & k^+ \leftrightarrow \bar{k}^+,\ k^0 \leftrightarrow \bar{k}^0,\ \pi^+ \leftrightarrow \pi^-,\ \pi^0 \leftrightarrow \pi^0,\ \gamma \to -\gamma, \\ R':\quad & p \leftrightarrow \Xi^-,\ n \to \Xi^0,\ \Lambda \leftrightarrow \Lambda,\ \Sigma^+ \leftrightarrow \Sigma^-,\ \Sigma^0 \leftrightarrow -\Sigma^0, \\ & k^+ \leftrightarrow \bar{k}^+,\ k^0 \to -\bar{k}^0,\ \pi^+ \leftrightarrow \pi^-,\ \pi^0 \leftrightarrow -\pi^0,\ \gamma \to -\gamma, \\ R'':\quad & p \leftrightarrow \Xi^-,\ n \leftrightarrow \Xi^0,\ \Lambda \leftrightarrow -\Lambda,\ \Sigma^+ \leftrightarrow \Sigma^-,\ \Sigma^0 \leftrightarrow -\Sigma^0, \\ & k^+ \leftrightarrow \bar{k}^+,\ k^0 \leftrightarrow -\bar{k}^0,\ \pi^+ \leftrightarrow \pi^-,\ \pi^0 \leftrightarrow -\pi^0,\ \gamma \leftrightarrow -\gamma. \end{aligned}$$

[16] R. S. McKean, Phys. Rev. **125**, 1399 (1962); N. Hoshizaki, S. Otsuki, W. Watari, and M. Yonezawa, Progr. Theoret. Phys. (Kyoto) **27**, 1199 (1962); S. Sawada, T. Ueda, W. Watari, and M. Yonezawa, *ibid.* **28**, 991 (1962).

[17] Y. Hara, Progr. Theoret. Phys. (Kyoto) **27**, 429 (1962); S. Minami, Y. Miyamoto, and N. Masuda, *ibid.* **27**, 846 (1962); R. J. N. Phillips, Phys. Letters **3**, 21 (1962).

TABLE I. Values of vector coupling constant of ω meson to the nucleon.[a]

Authors	Methods	$f_{\omega NN}^2/4\pi$
Sakurai	LS term in $N-N$ scattering	~ 10
Hara	$N-N$ forward dispersion relations	~ 9
Minami *et al.*	$N+\bar{N} \rightarrow 2\omega$ reaction	$5\sim 7$
McKean	$N-N$ scattering	~ 25
Hoshizaki *et al.*	$N-N$ scattering	$2\sim 12$
Phillips	$N-N$ and $N-\bar{N}$ scattering with Regge hypothesis	~ 10

[a] See Refs. 16 and 17.

should not be taken at its face value.[18] Anyhow, with this tentative value of the coupling constant, the partial width of ω meson into three pions now can be estimated by applying simple perturbation theory, which leads to the following result[19]:

$$\begin{aligned} \Gamma(\omega \rightarrow 3\pi) &\approx 1 \times (f_{\omega NN}^2/4\pi)\ \text{MeV}, \\ &\approx 5\sim 10\ \text{MeV}. \end{aligned} \tag{2}$$

Now, let us divide our discussions into two possible cases: (A) $\Gamma(\omega \rightarrow 3\pi) \gtrsim 5\sim 10$ MeV and (B) $\Gamma(\omega \rightarrow 3\pi) \lesssim 1$ MeV. In case of (A) which seems to be suggested by the present experiments,[7] one may argue that in view of the result (2) this decay process should be considered to be normal or allowed under some kinds of symmetry in the sense that its decay rate is approximately in accordance with that expected by ordinary perturbational considerations. On the other hand, in case of (B), we conclude that this decay process will be a forbidden one.

In either case, these requirements can be accomplished again by assuming R-conjugation invariance and transforming the ω meson under R conjugation as follows:

$$\begin{array}{lll} & \text{Case (A)} & \text{Case (B)} \\ R\ \text{inv:} & \omega \rightarrow -\omega, & \omega \rightarrow +\omega, \\ R'\ \text{or}\ R''\ \text{inv:} & \omega \rightarrow +\omega, & \omega \rightarrow -\omega, \end{array} \tag{3}$$

If we wish to accommodate this result with Sakurai's vector meson theory,[20] it will be natural to assume that, in case of (A), the ω meson is R invariantly associated with hypercharge current, or R' or R'' invariantly associated with baryonic charge current. In case of (B), the situation is just reversed. We have not yet succeeded in discriminating definitely between R invariance and R' or R'' invariance in strong interaction but if we recall the preceding discussions on $\gamma-3\pi$ interaction in Sec. 2, we may expect that R' or R'' invariance is more favorable than the usual R invariance.[21] Furthermore, if we actually assume R' or R'' invariance we conclude that, within Sakurai's vector meson theory the ω meson is preferably associated with baryonic charge current, since recent measurements on ω meson width suggest that we take assumption (A).

Now we proceed to the φ-meson width problem. As was remarked before, observations seem to suggest a rather small partial width of the φ meson into three pions. Therefore, we tentatively suppose that this decay process will be a forbidden one.

Then exactly the same reasoning which was applied to the ω-meson decay problem leads to the following assumption on the R-transformation property of the φ meson:

$$\begin{array}{ll} R\ \text{inv:} & \varphi \rightarrow +\varphi, \\ R'\ \text{or}\ R''\ \text{inv:} & \varphi \rightarrow -\varphi. \end{array} \tag{4}$$

This suggests that the φ meson is R invariantly associated with baryonic charge current, or R' or R'' invariantly associated with hypercharge current. In particular, if we assume R' or R'' invariance, the partial width $\Gamma(\varphi \rightarrow 3\pi)$ can be calculated to be[22]

$$\begin{aligned} \Gamma(\varphi \rightarrow 3\pi) &\approx 5 \times \beta \times (f_{\varphi NN}^2/4\pi)\ \text{MeV}, \\ &\approx 0.5 \times (f_{\varphi NN}^2/4\pi)\ \text{MeV}, \end{aligned} \tag{5}$$

which is not inconsistent with the present experimental data if we assume $f_{\varphi NN}^2/4\pi \approx 1\sim 2$.

It will be noted that, in this case, a mixing effect between ω and φ mesons is induced by R-conjugation violating interaction, since ω and φ mesons have different transformation properties under R conjugation.

4. DISCUSSIONS AND PREDICTIONS

From the foregoing considerations alone, it would be premature to draw definite conclusions on the symmetries of strong interaction, but it is our strong feeling that present experimental data on ω and φ meson widths are consistent with the assumption that ω and φ mesons are R' or R'' invariantly associated with baryonic charge and hypercharge currents, respectively, and that their coupling constants fall around $f_{\omega NN}^2/4\pi \approx 5\sim 10$ and $f_{\varphi NN}^2/4\pi \approx 1\sim 2$.[23] In this model, ω and φ

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[18] This is because the analyses thus far attempted do not properly take φ meson effect into account. However, if the partial width $\Gamma(\varphi \rightarrow 3\pi)$ actually turns out to be small as is shown by the present experimental data (Ref. 8), then these analyses would remain to be true. See also discussions given below.

[19] C. Itzykson, M. Jacob, F. Pham, and W. Alles, Phys. Letters **1**, 90 (1962).

[20] J. J. Sakurai, Ann. Phys. (N. Y.) **11**, 1 (1960).

[21] See Ref. 6.

[22] It should be noticed that the matrix element of $\varphi \rightarrow 3\pi$ has the same structure as that of $\gamma \rightarrow 3\pi$ transition matrix if we confine ourselves to the lowest order of $f_{\varphi NN}^2$ and fine structure constant e^2, respectively, and also to the local interactions.

[23] This value of $f_{\varphi NN}^2/4\pi$ is approximately in agreement with the prediction based on the SU(3) model if the φ meson is assumed to be the $T=0$ partner of the vector meson octet. A similar model to ours has been proposed and discussed in detail by Dashen and Sharp, Phys. Rev. **133**, B1585 (1964). The author is grateful to them for sending their results prior to publication. On the other hand, an attempt was made by S. Furui, S. Furuichi, S. Ishida, S. Kawasaki, and M. Sawamura, Progr. Theoret. Phys. (Kyoto) **30**, 840 (1963) in order to formulate a dynamical theory of composite particles based on the Sakata model. They obtained the effective coupling constants $f_{\omega NN}^2$ and $f_{\varphi NN}^2$, which are not inconsistent to the values given in this paper.

mesons are distinguished by R-conjugation parity and the mass difference between them should have the same origin as that between nucleon and cascade particle.

Now let us accept this hypothesis for the moment. Then, based on this model, we can make some specific predictions for decay and production phenomena of these vector mesons, which we shall list below.

(*i*) *Decay of the* ω *meson.* According to (2), the width of the ω meson into three pions $\Gamma(\omega\rightarrow 3\pi)$ is expected to be several MeV. It would be difficult to understand if a very small width was obtained under this hypothesis. A recent experiment[24] seems to support this hypothesis.

(*ii*) *Decay of the* φ *meson.* On the other hand, the partial decay width of the φ meson into three pions $\Gamma(\varphi\rightarrow 3\pi)$ should be very small. Moreover, since the φ meson is associated with the hypercharge current, the coupling constant $f_{\varphi KK}$ is equal in magnitude to $f_{\varphi NN}$ and, together with the result (5), we obtain following values for the partial width of φ meson into $K+\bar{K}$, $\Gamma(\varphi\rightarrow K+\bar{K})$, and the branching ratio between them:

$$\begin{aligned}\Gamma(\varphi\rightarrow K+\bar{K})&\approx 2\times(f_{\varphi KK}{}^2/4\pi)\ \text{MeV},\\ &\approx 2\sim 4\ \text{MeV}\end{aligned}\tag{6}$$

and

$$[\Gamma(\varphi\rightarrow 3\pi)/\Gamma(\varphi\rightarrow K+\bar{K})]\approx 0.2\sim 0.3,\tag{7}$$

both of which are not inconsistent with experiment. It should be noted that unitary symmetry model also predicts $\Gamma(\varphi\rightarrow K+\bar{K})\approx 2\sim 3$ MeV assuming that the φ meson belongs to the $T=0$ partner of vector meson octet.[25] The fact that the prediction based on the SU(3) model and our result [Eq. (6)] roughly agree with each other might imply that the experimental data on $\Gamma(\varphi\rightarrow K+\bar{K})$ do not necessarily indicate a strong support for the unitary symmetry model. On the other hand, we would like to emphasize that experimental determination of the ratio (7) would provide a useful test on various symmetry models.

(*iii*) *Production of the* ω *and* φ *mesons.* Since $f_{\omega NN}{}^2 \gg f_{\varphi NN}{}^2$, we can expect that in $\pi-N$ or $N-N$ collisions, ω mesons are copiously produced compared with φ mesons. More quantitatively, we estimate the production ratio as

$$\begin{aligned}\frac{\sigma(\varphi)}{\sigma(\omega)}\approx\frac{f_{\varphi\rho\pi}{}^2}{f_{\omega\rho\pi}{}^2}&\approx\beta\frac{f_{\varphi NN}{}^2}{f_{\omega NN}{}^2}\\ &\approx 10^{-2}\end{aligned}\tag{8}$$

FIG. 2. In both diagrams (A) and (B), ω and φ mesons are assumed to make a transition into γ through $B\bar{B}$ states. Here $B\bar{B}$ states stand for baryon pair states (pairs of nucleon and cascade particles).

in energy regions high enough so that peripheral collisions will dominate and the mass difference between ω and φ mesons can be neglected. This ratio (8) is not in disagreement with recent observations.[26]

(*iv*) *Isoscalar form factors.* The ω-meson contribution to the isoscalar form factor of the nucleon was discussed in detail before.[27] It was shown there that the contribution from ω meson actually gives rise to a reasonable size of isoscalar charge radius if we assume $\Gamma(\omega\rightarrow 3\pi)\lesssim 7$ MeV. In this paper, we want to show that φ-meson contribution is expected to be less important than the ω-meson contribution. The reason is as follows: In the pole approximation, their contributions to the isoscalar form factors come from two physically distinctive processes, $N\bar{N}\rightarrow\omega$ or φ and ω or $\varphi\rightarrow\gamma$. The former process is directly proportional to the coupling constants $f_{\omega NN}$ and $f_{\varphi NN}$. In the latter process, the transition $\omega\rightarrow\gamma$ is forbidden by R' or R'' invariance, while the transition $\varphi\rightarrow\gamma$ is an allowed process. Therefore, the ratio of these coupling constants, which are defined by $em_\omega{}^2/f_\omega$ and $em_\varphi{}^2/f_\varphi$, may be estimated as (see Fig. 2)

$$f_\varphi/f_\omega\approx\sqrt{\beta}\times m_\varphi{}^2/m_\omega{}^2\times f_{\omega NN}/f_{\varphi NN},\tag{9}$$

from which we obtain

$$\begin{aligned}\langle r^2\rangle_\omega:\langle r^2\rangle_\varphi&=\frac{1}{m_\omega{}^2}\frac{f_{\omega NN}}{f_\omega}:\frac{1}{m_\varphi{}^2}\frac{f_{\varphi NN}}{f_\varphi}\\ &\approx\sqrt{\beta}\times\left(\frac{m_\varphi}{m_\omega}\right)^4 f_{\omega NN}{}^2:f_{\varphi NN}{}^2\\ &\approx 3\sim 7:1.\end{aligned}\tag{10}$$

This shows that within a pole approximation φ-meson contribution to the isoscalar charge radius is unimportant compared to the ω-meson contribution.

(*v*) *Rare decay modes of* ω *and* φ *mesons.* We estimate the ratios of various rare decay modes of ω and φ mesons following the methods given in Refs. 9, 10,

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[24] N. Gelfand, D. Miller, M. Nussbaum, J. Ratan, J. Schultz *et al.*, Phys. Rev. Letters **11**, 436 (1963). See also J. J. Murry Jr., M. Ferro-Luzzi, D. O. Huwe, J. B. Shafer, F. T. Sulmitz, and M. Lynn Stevenson, Phys. Letters **7**, 388 (1963).

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[26] C. Alff, D. Berley, D. Colley, N. Gelfand, U. Nauenberg, *et al.*, Phys. Rev. Letters **9**, 322 (1962); M. Abolins, R. L. Lander, W. A. W. Mehlhop, N. Xuong, and P. M. Yager, *ibid.* **11**, 381 (1963).

[27] K. Kawarabayashi and A. Sato, Progr. Theoret. Phys. (Kyoto) **28**, 173 (1962); **28**, 667 (1962). See also Ref. 10.

28, and 29 as follows:

$$\begin{aligned}\frac{\Gamma(\varphi\rightarrow 2\pi)}{\Gamma(\omega\rightarrow 2\pi)} &= \frac{f_\omega^2}{f_\varphi^2}\frac{m_\varphi}{m_\omega}\left|\frac{F_\pi(m_\varphi^2)}{F_\pi(m_\omega^2)}\right|^2\left(\frac{1-(4\mu^2/m_\varphi^2)}{1-(4\mu^2/m_\omega^2)}\right)^{3/2}\\ &\approx\beta^{-1}\frac{f_{\varphi NN}^2}{f_{\omega NN}^2}\left(\frac{m_\omega}{m_\varphi}\right)^3\left|\frac{F_\pi(m_\varphi^2)}{F_\pi(m_\omega^2)}\right|\left(\frac{1-(4\mu^2/m_\varphi^2)}{1-(4\mu^2/m_\omega^2)}\right)^{3/2} \qquad (11)\\ &\approx 0.04,\end{aligned}$$

$$\begin{aligned}\frac{\Gamma(\varphi\rightarrow\pi^0+\gamma)}{\Gamma(\omega\rightarrow\pi^0+\gamma)} &= \frac{f_{\varphi\rho\pi}^2}{f_{\omega\rho\pi}^2}\left(\frac{m_\varphi}{m_\omega}\right)^3\left(\frac{1-(\mu^2/m_\varphi^2)}{1-(\mu^2/m_\omega^2)}\right)^3\\ &\approx\beta\frac{f_{\varphi NN}^2}{f_{\omega NN}^2}\left(\frac{m_\varphi}{m_\omega}\right)^3\left(\frac{1-(\mu^2/m_\varphi^2)}{1-(\mu^2/m_\omega^2)}\right)^3 \qquad (12)\\ &\approx 0.05,\end{aligned}$$

and

$$\begin{aligned}\frac{\Gamma(\varphi\rightarrow l^+ + l^-)}{\Gamma(\omega\rightarrow l^+ + l^-)} &= \frac{f_\omega^2}{f_\varphi^2}\frac{m_\varphi}{m_\omega}\\ &\approx\beta^{-1}\frac{f_{\varphi NN}^2}{f_{\omega NN}^2}\left(\frac{m_\omega}{m_\varphi}\right)^4 \qquad (13)\\ &\approx 1,\end{aligned}$$

where we have adopted the values $f_{\omega NN}^2/f_{\varphi NN}^2\approx 5$ and $\beta\approx 0.1$ and neglected the lepton mass in the last expression (13).

It will be noted that all these ratios are small except the last one, which will make it difficult to observe the rare decay modes of the φ meson.

Finally, we would like to add a remark on a paper by Bronzan and Low[30] in which they introduced a new selection rule for boson systems and discussed some consequences of this selection rule. The conclusions obtained by them, however, are formally equivalent to ours as far as boson systems are concerned. We do not want to describe all the consequences of R' or R'' invariance in this paper, since these are already discussed in Ref. 6. In any case, a crucial test of R-conjugation symmetry will be primarily a precise measurement of the anomalous magnetic moment of the Λ hyperon μ_Λ, which, according to R-conjugation invariance, should vanish or at least should be very small.

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